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# HARMONIC ANALYSIS OF THE LONG-TERM COINTEGRATION BETWEEN PETKIM STOCK PRICES AND USD/TL EXCHANGE RATES

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**ABSTRACT**

*This study employs harmonic analysis to investigate the relationship between Petkim stock prices and the USD/TL exchange rate. The harmonic analysis is a robust analytical method in modern time series analysis, particularly suitable for data exhibiting stationary characteristics without any apparent trend. This approach emphasizes the importance of considering both linear, nonlinear, and periodic components to fully understand and predict the dynamics of the financial market, demonstrating significant implications for econometric modeling. The dynamics between stock prices and exchange rates attract attention due to their profound impact on economic indicators and investment strategies in the volatile world of financial markets. Our study covers the period from May 1, 2023, to March 29, 2024, marked by significant economic events and policy changes potentially affecting Turkey's capital and currency markets. Consequently, a model with a parabolic (quadratic) trend for both factors and small amplitude sinusoidal and cosinusoidal departures around this trend has been constructed.*

## 1. Introduction

During the period under study, several significant economic and political events impacted Petkim stock prices and the USD/TL exchange rate. One of these was the adjustments in monetary policy by the Central Bank of the Republic of Turkey, particularly changes in interest rates aimed at curbing inflation, which affected investor confidence and market liquidity. These changes directly influenced the USD/TL exchange rate and indirectly impacted financial markets, including Petkim stocks. Petkim's operations in the petrochemical sector have shown high sensitivity to global oil prices. During the monitored period, volatility in oil prices was observed due to OPEC's revenue decisions and geopolitical tensions in oil-producing regions. This volatility affected Petkim's operational costs and profitability, subsequently influencing its stock prices. Additionally, Turkey's high inflation rates led to the depreciation of the Turkish lira, increasing the costs of imported raw materials and reducing the global competitiveness of Petkim's products. Depreciation also affected financial transactions related to foreign trade and investment flows, causing fluctuations in the USD/TL exchange rate.

## 2. Analysis of recent publications

In the paper, the initial conditions for the manifestation of cointegration approaches amid fluctuations in the AZN/TRY and USD/TL exchange rates during the sharp depreciation of the Turkish lira in the first half of 2023 were examined (Burjaliyeva, 2024). The study employed modern econometric methodologies, including the Johansen cointegration test, Granger causality test, vector error correction, and other relevant approaches. In this article, the mathematical model of the financial market in continuous time with models of stochastic moving technical dynamic systems is constructed, where the coordinates of this vector are established from the vector-matrix differential equation of the state vector of the financial market (Оруджев, 2009). The article studied the properties of trajectories, mathematical expectations, and dispersions of these trajectories with certain constraints on the coefficients of the differential system, and solves the system of differential equations for the covariance matrix. Additionally, stability and instability in markets, as well as explosive fluctuations leading to major financial disasters, were also addressed. The structural changes in the AZN/RUB and USD/RUB exchange rates amid increasing sanctions against Russia from January 2, 2023, to September 6, 2023 (five-day indicators, 178 observations), were examined considering the error correction mechanism (ECM) and autoregressive distributed lag (ARDL) models (Orudzhev and Mamedova, 2024). The precise specification of regression models confirming the statistically weak significant co-directional impact of changes in the USD/RUB exchange rate on the AZN/RUB exchange rate was determined. An econometric analysis of changes in the USD/AZN exchange rate based on real indicators from January 1, 2013, to January 10, 2017, was conducted, with their endogenous variability obtained through empirical analysis (Orudzhev et al., 2018). The most suitable model for exchange rate dynamics was constructed using computer modeling, mean-square error indicators of convergence, and Fourier series approach with mean-square divergence and time-dependent behavior in the time series (Orudzhev and Mamedova, 2020). This study was based on 360 daily observations of EUR/AZN currency exchanges from February 3, 2017, to August 3, 2018.

Harmonic analysis is widely used in technical fields, but it is unusual in economics and finance, where ARIMA and GARCH modeling are more commonly employed. The identification of hidden periodic components in high-frequency financial data using harmonic analysis was studied here, with the example of foreign exchange rates (Dolinar, 2013). Statistical analysis methods necessary for constructing double regression models (variance analysis, correlation-regression analysis, statistical assumptions in data analysis) for theoretical analysis of forecast indicators (Hall, 1992), error variance decomposition, and modeling calculations, as well as two-dimensional vector autoregression models and cointegration in these models, approaches to modern economic and mathematical modeling (Verbeek, 2012; Orudzhev, 2018), EXCEL software packages (Воскобойников, 2008) and Eviews-12 software package (Матюшок et al., 2011) were used. The study examined the impact of global crises, including the Global Financial Crisis (GFC), the COVID-19 pandemic, and the subsequent inflation crisis, on changes in the Real Effective Exchange Rate (REER) (Mierzejewski and Prażmowski, 2024). The periodicity of the REER was investigated as influenced by economic shocks, highlighting how the economic disruptions caused by the pandemic shaped REER dynamics differently from the more financially driven fluctuations of the GFC. The methodology used a comparative harmonic analysis approach, utilizing time series data to track REER movements across various countries. Findings indicate that the GFC and COVID-19 pandemic shortened the periods of cyclicality. Additionally, there was a notable improvement in the synchronization of REER movements post-GFC, suggesting that economies may converge in their responses to global economic shocks. This convergence implied potential stabilization of exchange rate movements in future crises, emphasizing the importance of coordinated monetary policy.

## 3. Methodology

Harmonic analysis is a methodology used to analyze periodic changes in time series of variables. This analysis aims to identify seasonal or cyclical components within the data. The basic idea of the Fourier series is to approximate a periodic function as a combination of simple oscillating functions, particularly sines and cosines.

$$\hat{y}_t = a_0 + \sum_{k=1}^{m}(a_k \cos kt + b_k \sin kt),\ \ t=1,2,...,n \quad (1)$$

Here, $\hat{y}_t$ represents the value of the trend function at time (t) in the model constructed for the time series using harmonic analysis. The parameter (k) denotes the number of the harmonic in the Fourier series, (m) represents the total number of harmonics, (n) indicates the total number of observations in the series, and (t) is the time variable, which can take values such as $0, \frac{2\pi}{n}, \frac{2*2\pi}{n}, \ldots, \frac{(n-1)*2\pi}{n}$.

The parameters of the model given in (1) can be calculated using the Ordinary Least Squares (OLS) method. By applying this method, a system of (2m+1) normal equations is obtained, from which the following estimation expressions can be derived through simple algebraic calculations:

$$a_0 = \bar{y}_t,\ \ (t=1,2, \ldots n) \quad (2)$$

$$a_k = \frac{2}{n}\sum_{t=1}^{n} y_t \cos kt,\ k=0,1,.., \frac{n}{2} \quad (3)$$

$$b_k = \frac{2}{n}\sum_{t=1}^{n} y_t \sin kt,\ k=0,1,\ldots, \frac{n}{2} \quad (4)$$

After estimating the parameters, it is determined which harmonic best describes the harmonic variation of the time series. Increasing the number of harmonics improves the accuracy of the approximation, but this can also increase the value of the quadratic deviation. An increase in quadratic deviation reduces the significance of the model. The mean square deviation is used to indicate how much the observed indicators deviate from the average level of the series. The smaller the mean approximation error and the mean square deviation for the given series, the more adequate the constructed model will be. The mean square deviation is calculated using the following formula:

$$\sigma_{y_t} = \sqrt{\frac{\sum_{t=1}^{n}(y_t-\hat{y}_t)^2}{n-m}} \quad (5)$$

The mean approximation error is determined using the following formula:

$$\bar{\epsilon} = \frac{1}{n}\sum_{t=1}^{n} \left| \frac{y_t-\hat{y}_t}{y_t} \right| \cdot 100\% \quad (6)$$

The main computational formula of harmonic analysis can also be written in the following forms:

$$\hat{y}_t = a_0 + \sum_{k=1}^{m} c_k(\cos kt + \varphi_k) \quad (7)$$

or

$$\hat{y}_t = a_0 + \sum_{k=1}^{m} c_k(\sin kt + \varphi_k) \quad (8)$$

Here, $c_k$-is the amplitude of the constructed model, $\varphi_k$-is the phase of the model.

Amplitude

$$c_k = \sqrt{a_k^2 + b_k^2} \quad (9)$$

Phase

$$\varphi_k = \text{arctg}\left(\frac{-b_k}{a_\kappa}\right) \quad (10)$$

is calculated as follows.

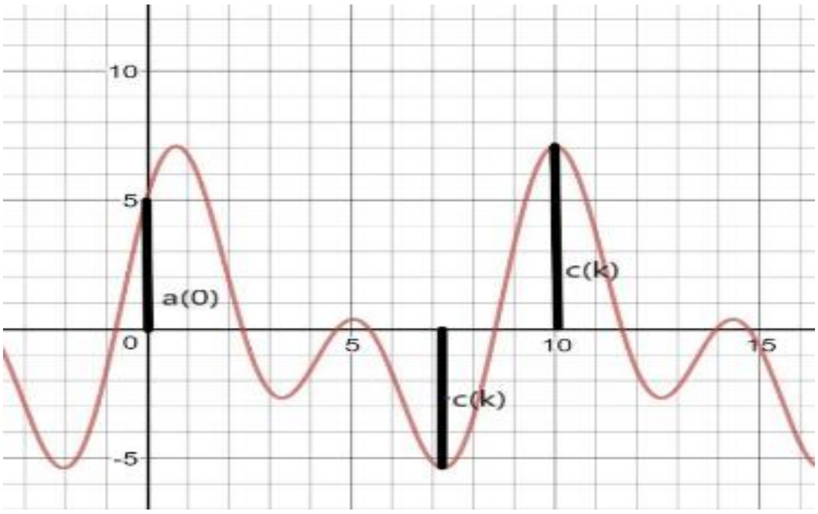


**Fig.1** Description of phase and amplitude

The amplitude indicates how far the graph of the model constructed using harmonic analysis is from the ( t ) (abscissa) axis (graph 1). The distribution of the amplitude of the harmonic components of the signal by frequencies is the amplitude spectrum, while the corresponding distribution of phases is the phase spectrum.

## 4. The main results of the study

The descriptive statistics of the factors studied are presented in the table below

**Table 1. Descriptive statistics**

| | PETKIM_PRICE | USD/TL |
|---|---|---|
| Mean | 19.18921 | 27.39615 |
| Median | 19.75000 | 27.78490 |
| Maximum | 25.44000 | 32.44910 |
| Minimum | 11.83000 | 19.44950 |
| Std. Dev. | 3.449713 | 3.373346 |
| Skewness | -0.405601 | -0.959881 |
| Kurtosis | 2.308189 | 3.392066 |
| Jarque-Bera | 11.36653 | 38.39199 |
| Probability | 0.003402 | 0.000000 |
| Sum | 4605.410 | 6575.075 |
| Sum Sq. Dev. | 2844.224 | 2719.691 |
| Observations | 240 | 240 |

The descriptive statistics indicate that both factors exhibit left-skewed asymmetry. However, the asymmetry is weak in Petkim, whereas it is significant in USD/TL. The kurtosis values satisfy the 3±1 conditions, indicating a distribution close to normal. However, the Jarque-Bera test rejects the normality assumption.

We apply harmonic analysis to the growth rate of Petkim stock prices and changes in the USD/TL exchange rate. Daily data from May 1, 2023, to March 29, 2024, comprising 240 observations for each parameter, are used [13-14] .

First, let's examine the single harmonic model for each parameter, which corresponds to ( m = 1 ). The movement dynamics and smoothing curve depicted in figure 2 can be obtained using the instrumentation procedures for MS EXCEL.

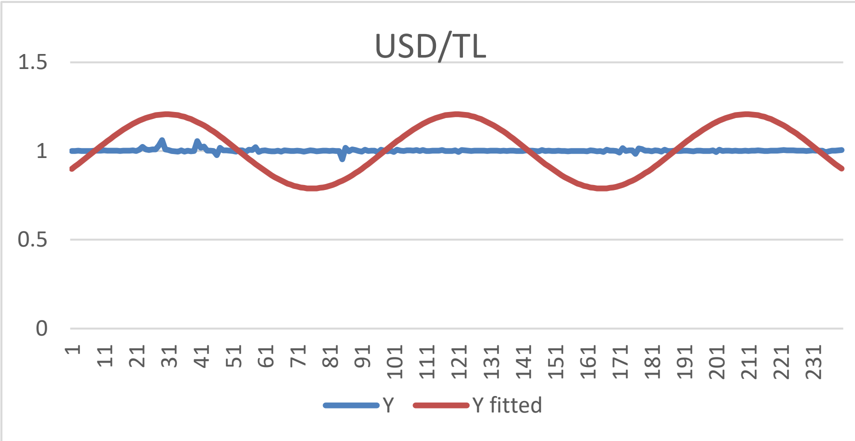


**Fig 2.** USD/TL exchange rate motion dynamics and smooth curve in the case of one harmonic

Based on the graph, the analytical form of the trend function for USD/TL is depicted below:

$$USD/TL_t = 1 - 0.111cos(t) + 0.177sin(t) \quad (11)$$

Here, for USD/TL, the mean square deviation is σ=0.1414, and the mean error of approximation is $\bar{\varepsilon}$=12.53%.

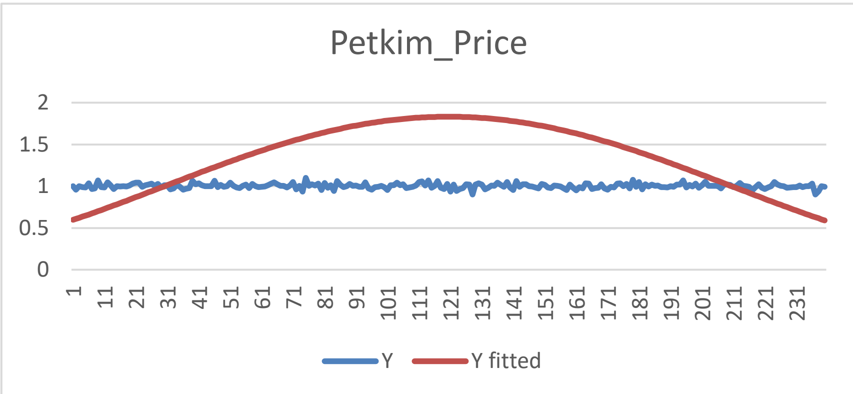


**Fig. 3** Petkim stock price motion dynamics and smooth curve in the case of one harmonic

From the graphical representation, the analytical form of the trend function for Petkim stock prices is as follows:

$$PETKIM\_PRICE_t = 1 - 0.418cos(t) + 0.716sin(t) \quad (12)$$

For Petkim, the mean square deviation is σ = 0.3951, and the mean approximation error is $\bar{\varepsilon}$= 44.7%. Now, let's consider the two-harmonic model, i.e., the case where (m = 2). The dynamic description can be obtained by performing the relevant algorithmic procedures in the EXCEL software package, as shown in figures 2 and 3.

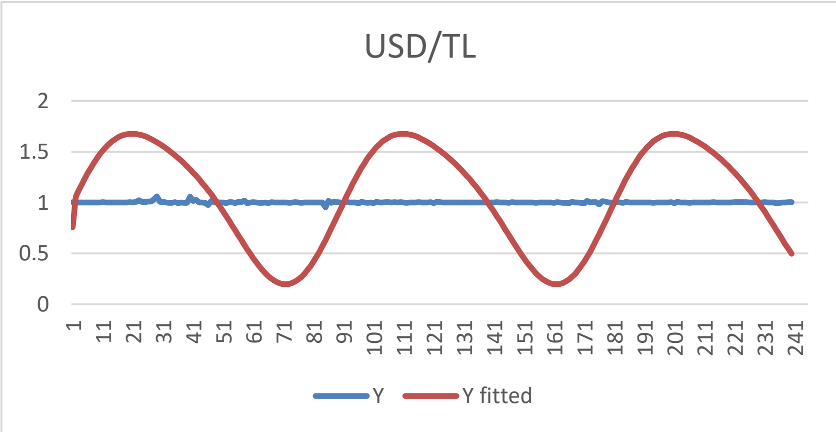


**Fig. 4** USD/TL exchange rate motion dynamics and smooth curve in the case of two harmonics

The analytical form of the model is shown as the following trigonometric expression:

$$USD/TL_t = 1 - 0.111cos(t) + 0.045cos2(t) + 0.715sin(t) + 0.092sin2(t) \quad (13)$$

For USD/TL, the root mean square deviation is σ=0.4959, and the mean error of approximation is $\bar{\varepsilon}$=44.58%.

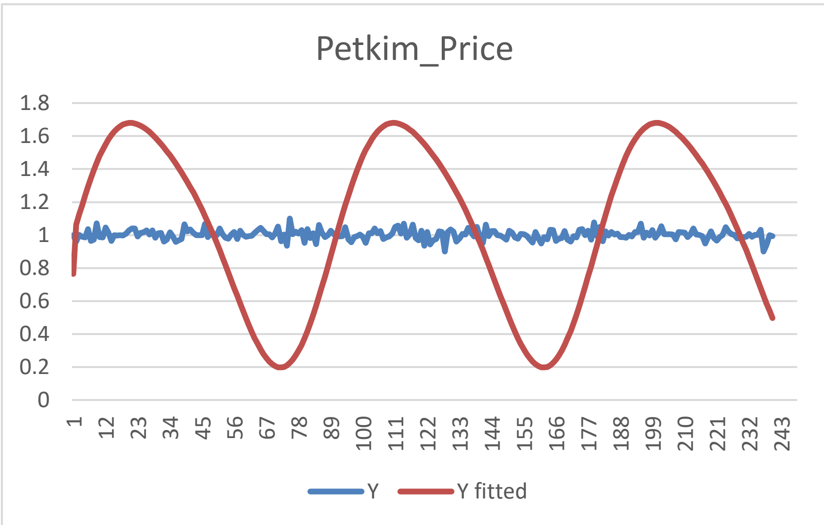


**Fig. 5** Petkim stock price motion dynamics and smooth curve in the case of two harmonics

$$PETKIM\_PRICE_t = 1 - 0.108cos(t) + 0.043cos2(t) + 0.715sin(t) + 0.094sin2(t) \quad (14)$$

For Petkim, the mean square deviation σ=0.4955, and the mean error of approximation, $\bar{\varepsilon}$=44.62%.

Now let's look at the case of m = 3.

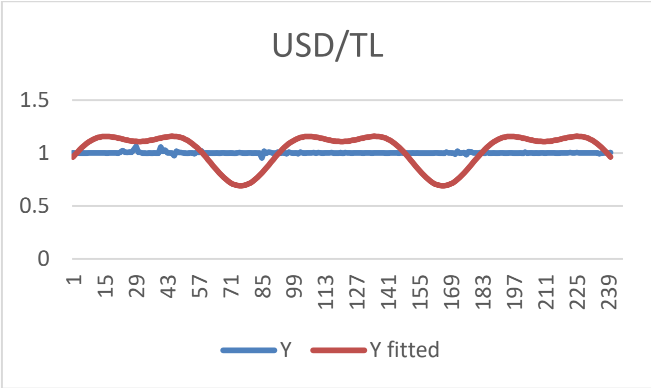


**Fig. 6** USD/TL exchange rate motion dynamics and smooth curve in the case of three harmonics

Based on the graph, the analytic function of the trigonometric model is as follows.

$$USD/TL_t = 1 - 0.111\cos(t) + 0.045\cos2(t) + 0.001\cos t3(t) + 0.177\sin(t) + 0.092\sin2(t) + (-6.2E - 05)\sin3(t) \quad (15)$$

For USD/TL, the mean square deviation σ= 0.15184, and the mean error of approximation, $\bar{\varepsilon}$= 13.64917%.

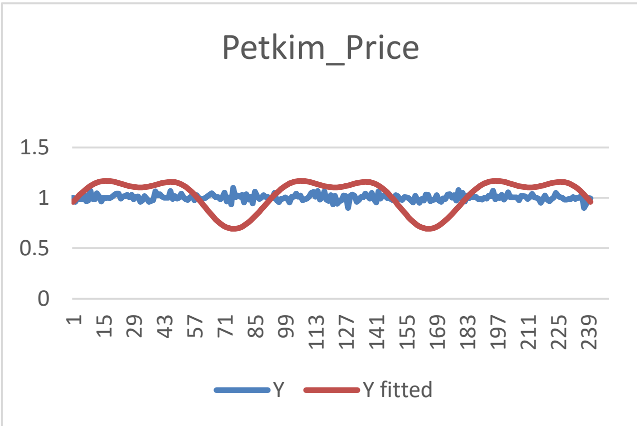


**Fig. 7** Petkim stock price motion dynamics and smooth curve in the case of three harmonics

Based on the graph, the analytic function of the trigonometric model is as follows.

$$PETKIM\_PRICE_t=1-0.108\cos(t)+0.043\cos2(t)-0.002\cos t3(t)+0.179\sin(t)+0.094\sin2(t)-0.006\sin3(t) \quad (16)$$

For Petkim, the mean square deviation σ= 0.1545, and the average error of approximation, $\bar{\varepsilon}$= 13.86055%.

Since the values of mean square deviation and approximation increase in the next harmonics, it is possible to be satisfied with the separation of only 3 harmonics.

It should be noted that the phase for each harmonic must be calculated separately. In harmonic analysis, phase angles (φ) represent the displacements or time differences of cyclic components relative to a sinusoidal reference. These phase changes are crucial for interpreting dynamic relationships and synchronizations between different time series, such as Petkim stock prices and USD/TL exchange rates. The econometric interpretation of the phase angles for both time series is provided below:

Phase for Petkim $\varphi_1$=1,042582rad, $\varphi_2$= -1,13818rad, $\varphi_3$=-1,12675rad

Phase for USD/TL $\varphi_1$= 1,010780141rad, $\varphi_2$= -1,11586rad, $\varphi_3$= 0,115983rad

Now, let's do a visual analysis of each indicator. A quadratic trend is the best trend, and small harmonic oscillations of the sin and cos type are made around this parabola.

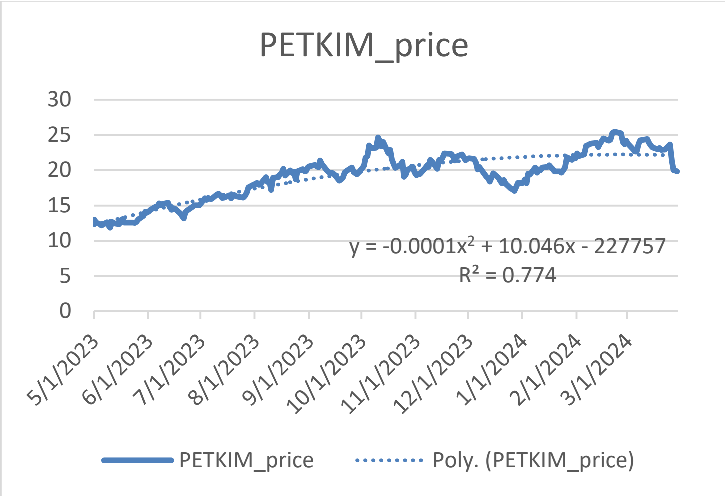


**Fig. 8** Dynamics of Petkim Stock Prices

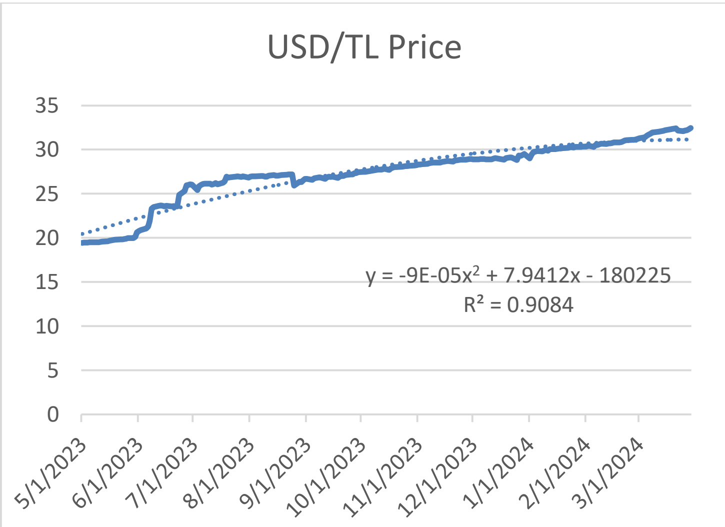


**Fig. 9** Dynamics of USD/TL

In USD/TL dynamics, the best approximation of the trend is the quadratic trend. The general dynamics are small harmonic oscillations around this trend.

As a result,

$$USD/TL_t = -9E - 05x^2 + 7{,}9412x - 180225 + 1 - 0.111\cos(t) + 0.045\cos2(t) + 0.001\text{cost}3(t) + 0.177\sin(t) + 0.092\sin2(t) + (-6.2E - 05)\sin3(t) \quad (17)$$

$$PETKIM_t = 0{,}0001x^2 + 10{,}046x227757 + 1 - 0.108\cos(t) + 0.043\cos2(t)0.002\text{cost}3(t) + 0.179\sin(t) + 0.094\sin2(t) - 0.006\sin3(t) \quad (18)$$

Obtained formulas can be applied in trigonometric cointegration analysis and ECM model construction.

## Conclusion

This study applied harmonic analysis to explore the long-term cointegration relationship between Petkim stock prices and the USD/TL exchange rate, emphasizing the periodic components that influence the dynamics of these financial variables. By modeling both a parabolic trend and small harmonic deviations, we were able to capture the underlying cyclical behaviors that traditional methods often overlook.

The results demonstrated that the relationship between Petkim stock prices and USD/TL exchange rates is characterized by three primary harmonics, which provide a sufficiently accurate representation of the cyclical fluctuations in both variables. The harmonic analysis revealed that despite significant external factors, such as fluctuations in oil prices, inflation, and monetary policy changes the long-term relationship between these variables remains stable.

The implications of this research are significant for both financial analysts and policymakers. The identification of periodic patterns in stock prices and exchange rates offers a new dimension for forecasting, particularly in volatile markets like Turkey's, where economic indicators can change rapidly. Furthermore, the model's ability to incorporate both linear and nonlinear components makes it a robust tool for understanding the complex dynamics of financial markets. Thus, a model with a parabolic (quadratic) trend and 3-harmonic sine and cos deviations of small amplitude around them was built for both factors.

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